Space-time structure of a bound nucleon

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Distortions of bound nucleon space-time structure, measured in deep inelastic scattering off nuclei, are discussed from view-point of a theoretical approach based on the Bethe-Salpeter formalism. It is shown that modification of the structure function F_2^{N} results from relative time dependence in Green functions of bound nucleons. The modification plays fundamental role in analysis of deep inelastic scattering experiments and allows one to obtain new information about nucleon structure.

1. INTRODUCTION

Decades of matter structure studying has shown that complete information about nucleon structure cannot be obtained from free nucleon data only. First of all, it is connected with absence of a stable enough free neutron target. The attempts to solve this problem were based on utilization of nuclear targets. However, small nuclear effects, which were supposed to be negligible, led to the qualitatively different results for the structure function of a nucleon bound in a deuteron and in heavy nuclei [1]. This phenomenon reflects untrivial difference between nuclear and nucleon structure at internucleon distances and its evolution with atomic number A. Nature of the effect was analyzed in numerous models which were produced since it was discovered (comprehensive reviews of the models can be found in [2]). Summing up the basic qualitative pictures, proposed in the models, one can conclude that the difference between nucleon and nuclear structure functions should originate from properties of nucleon structure and structure of nucleon-nucleon interaction. Thus, from the one hand the nuclear effects should be defined by properties the n-nucleon Green functions ($n = 1, 2 \ldots A$), from the other hand detailed information about *nuclear* effects can provide new information about *nucleon* structure. In this paper the general properties of relativistic bound states, which presumably lead to the EMC-effect, are discussed. It is shown that the relativistic consideration allows one to establish new regularities in nuclear structure which are important in the construction of the nucleon parton distributions.

2. SPACE-TIME STRUCTURE OF RELATIVISTIC BOUND STATES

The amplitude of deep inelastic scattering is defined by the imaginary part of the forward Compton amplitude:

$$W_{\mu\nu}^A(P,q) = \mathrm{Im}_{q_0} i \int d^4 x e^{iqx} \langle A, P | \mathrm{T} \left(J_\mu(x) J_\nu(0) \right) | A, P \rangle$$
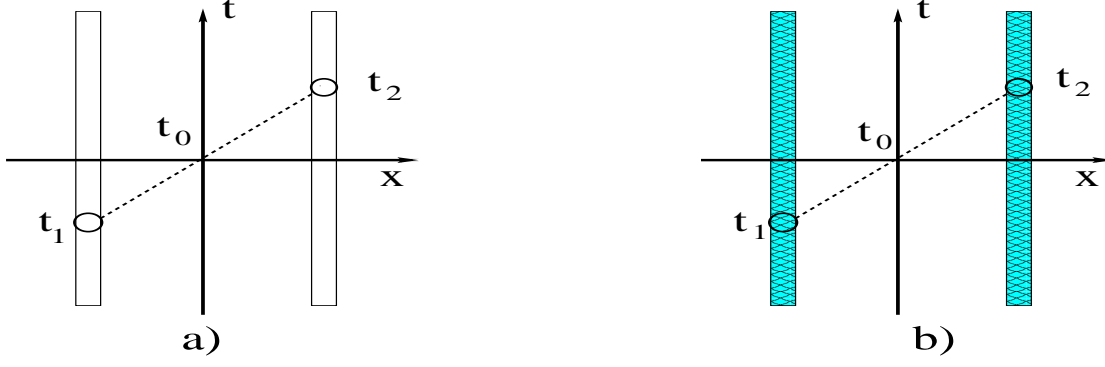
Figure 1. Schematic presentation of the space-time diagram for the two-nucleon bound state for the cases when the nucleon has a static (a) and dynamic (b) structure.

In the framework of the Bethe-Salpeter formalism the amplitude is expressed in terms of solutions of the homogeneous Bethe-Salpeter equation [3]

$$\chi_{\alpha,P}^{A}(\mathcal{X}) = \int d\mathcal{Z}d\mathcal{Z}' S_{(n)}(\mathcal{X}, \mathcal{Z})\overline{G}_{2n}(\mathcal{Z}, \mathcal{Z}')\chi_{\alpha,P}^{A}(\mathcal{Z}') \tag{1}$$

and the Mandelstam vertex $\overline{G}_{2(n+1)_{\mu\nu}}(\mathcal{Z}, x, \mathcal{Z}')$ [4]:

$$\langle A, P|\mathrm{T}(J_\mu(x)J_\nu(0))|A, P\rangle = \int d\mathcal{Z}d\mathcal{Z}'\overline{\chi}_{\alpha,P}^{A}(\mathcal{Z})\overline{G}_{2(n+1)_{\mu\nu}}(\mathcal{Z}, x, \mathcal{Z}')\chi_{\alpha,P}^{A}(\mathcal{Z}'). \tag{2}$$

The calligraphic letters denote a set of nucleons positions in the four-dimensional space – $\mathcal{Z} = z_1, \ldots z_n$. Relative positions of the nucleons are defined by four-dimensional intervals – $r_i = \sum_j^n z_j/n - z_i$. This is the most unusual feature of relativistic bound states that nucleons inside it are not only divided by three-dimensional intervals but also *shifted in time*. The shift is defined by the zero component of the interval – *relative time* $\tau_i = r_{i0}$.

If one considers nucleons as three-dimensional objects then the shift in time looks like unphysical feature [5]. One should assume in this case that the shift does not affect on observables, and different values of the variables τ_i in Eq. (1) should lead to equivalent quasi-potential approaches [5]. Recently it was shown, however, that the approaches are not equivalent from the point of view of relativistic covariance [6]. In general, the covariance can be kept only in equal-time approaches where $\tau_i = 0$. This contradiction points to existence of observable effects which could result from the shift in time of bound nucleons. Existence of such effects can be natural within the hypothesis that bound nucleons are shifted in time four-dimensional objects.

The space-time diagram for the bound state of two nucleons, which are considered to be four-dimensional objects, can be represented as it is shown in Figure 1. The Fermi motion is disregarded for simplicity. The static structure shown in diagram (a) is homogeneous in time, while diagram (b) represents an inhomogeneous in time bound nucleon structure. If one makes an instant flash of the bound state at the moment t_0 one finds that the structure is defined by one nucleon at the moment t_1 in the future and by another one at the moment t_2 in the past. If the nucleon had the static structure then the three points

t_0, t_1, t_2 in Figure 1(a) would be equivalent to each other and no effect would be observed from the shift in time. In another case, the points in Figure 1(b) are not equivalent to each other due to the evolution of the nucleon structure in time. The shift in time would then show up in modifications of the observables. From the stability of bound nucleons one can infer that the evolution of the nucleon structure in time has a periodic character. The period θ should be comparable to the mean value of relative time absolute $\theta \sim |\tau|$. If $|\tau| \gg \theta$ or $|\tau| \ll \theta$ then no effect from the shift in time of bound nucleons can be measured.

The established relation between the structure of the nucleon in the future and the measurements at present looks as if the causality principle is violated. However, the quantum nature of the considered objects does not allow one to perform an instant measurement. Instead one have to consider the contribution of nucleon states averaged between the time moments t_1 and t_2. This suggests a qualitative interpretation of the modification of the bound nucleon structure. One can estimate from the uncertainty relation the size of the area in which the nucleon can be localized in 4-space. The radius ρ of the area, "nucleon localization radius", can be related to the nucleon mass as follows: $\rho^2 = \Delta t^2 - \Delta x^2 \sim 1/m_{\mathrm{N}}^2$. The shift in time results in additional uncertainty $|\tau| = |t_2 - t_1|$ in definition of the time moment of the measurement: $\widetilde{\rho}^2 = (\Delta t + |\tau|)^2 - \Delta x^2 \sim 1/m_{\mathrm{N}}^{*2}$. Since $|\tau| > 0$ by definition, the localization radius becomes larger $\widetilde{\rho} > \rho$ and effective mass of the nucleon becomes smaller $m^* < m$. This is similar to the hypothesis of the x-rescaling model [7], where the Bjorken variable x of a bound nucleon is rescaled $\widetilde{x} = x/(1 - \epsilon/m_{\mathrm{N}})$ due to the shift of nucleon mass by the rescaling parameter ϵ: $m^* = m - \epsilon$. The x-rescaling model gives qualitative description of the deviation of nuclear to nucleon structure function ratio from unity (EMC – effect). Thus we can conclude that the shift in time manifests itself in observables, the EMC effect being one of the examples.

This conclusion has been justified in general terms in the publications [8,9] where the nuclear structure functions were calculated in the framework of the covariant approach based on the Bethe-Salpeter formalism. In the framework of this approach the relative time dependence has been consistently taken into account with the help of analytical properties of nucleon Green functions. The developed approach allows one to consider the theory with equal-time bound nucleons as the leading approximation in description of relativistic bound states, where the relativistic corrections, connected with the relative time dependence, are disregarded. In order to take the corrections into account, one has to keep the dependence on the relative time through the entire chain of calculation of observables. As a result, the nuclear structure function has been obtained in the form:

$$F_2^A(x_A) = \int \frac{d^3k}{(2\pi)^3} \sum_{a,a'}^{A-1} \left[\frac{E_a - k_3}{E_a} F_2^a(x_a) + \frac{\Delta_{a,a'}^A}{E_a} x_a \frac{dF_2^a(x_a)}{dx_a} \right] \Phi_{a,a'}^A(\mathbf{k})^2, \tag{3}$$

where the Bjorken variables for the nucleus A and the nuclear fragment a are introduced in the form $x_A = Q^2/(2P_A \cdot q)$ and $x_a = Q^2/(2p_a \cdot q)$. The function $\Phi_{a,a'}^A(\mathbf{k})^2$ defines distribution of the nuclear fragment a ($a = \mathrm{N}, \mathrm{D}, {}^3\mathrm{He}, \ldots$) in field of the spectator system a' ($a' = \mathrm{N}, \mathrm{NN}, \mathrm{D}, \mathrm{DN}, {}^3\mathrm{He}, \ldots$). The energy of the nuclear fragment is denoted as $E_a = \sqrt{M_a^2 + \mathbf{k}^2}$, M_a is mass of the fragment, the coefficients $\Delta_{\mathrm{a},\mathrm{a}'}^A = -M_A + E_{\mathrm{a}} + E_{a'}$ can be interpreted as the removal energy of the corresponding nuclear fragment. The term with derivative of the nucleon structure function in Eq.(3) results from the shift in time

of bound nucleons [9]. It is clear that the result can be rewritten in the form which one obtains within the x-rescaling model:

$$F_2^A(x_A) = \int dy d\epsilon F_2^N \left(\frac{x_A}{y - \epsilon/M_A} \right) \int \frac{d^3k}{(2\pi)^3} \frac{my}{E_N} \delta \left(y - \frac{E_N - k_3}{m} \right) \sum_{a'} \Phi_{N,a'}^A(\mathbf{k}) \delta \left(\epsilon - \Delta_{N,a'}^A \right).$$

The term with derivative in Eq.(3) leads to the depletion of the deuteron to nucleon structure functions ratio from unity [8] in numerical calculations. The evaluated ratio $F_2^A(x)/F_2^D(x)$ is in good agreement with the data available for $A = 4$. When evaluated at $A = 3$, the ratio offers the prediction for the experiments with ^{3}He, ^{3}H and D targets. On the basis on this prediction the two-stage conception of A-dependence for the nuclear structure functions was proposed. At the first stage ($1 \leq A \leq 4$) the pattern of the F_2^A/F_2^N changes due to the relative time effects. At the second stage the pattern does not change but the amplitude of ratio oscillation around unity grows due to nuclear density evolution [10]. The calculations based on this conception give good description of heavy nuclear data for the ratio [9].

3. CONCLUSION

To sum up the discussion presented in this paper, the following conclusions can be made. The proposed qualitative picture shows that the shift in time of bound nucleons exposes dynamical properties of a nucleon structure. Both the clear connection between the relativistic picture and the x-rescaling model and successful description of existent EMC effect data proves that the EMC effect is manifestation of the shift in time of the bound nucleons. The relativistic calculations show that the observable effects of the shift in time are defined by the derivative with respect to x of the nucleon structure functions, which, therefore, reflect the dynamical properties of a nucleon structure. Thus, precision data on nuclear to deuteron structure functions ratio will provide information about the derivative, what is important in construction of nucleon parton distributions.

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